International Journal of Theoretical Physics, Vol. 39, No. 2, 2000

Ground State for Fractional Quantum Hall Effect

A. Jellal[1,2]

Received June 22, 1999

We investigate the ground-state wave function for an explicit model of electrons in an external magnetic field with specific interparticle interactions. The excitation states of this model are also given.

The discovery of the fractional quantum Hall effect (FQHE) [1] has stimulated extensive studies on the two-dimensional quantum many-electron system in strong magnetic field. Considerable progress [2] has been made in understanding that the FQHE is essentially based on Laughlin's picture of the incompressible two-dimensional quantum fluid, which exhibits an energy gap [3].

The purpose of the present paper is to determine the wave functions of a Hamiltonian model involving, in addition to the quasicanonical momentum term, the specified interactions between the electrons. We also derive its excitation wave functions.

To begin, let us consider a system of interacting electrons moving in a plane and subjected to a perpendicular uniform magnetic field B. In the appropriately chosen system of units ($c = \hbar = m = e = 1$, $B = 2$) and symmetric gauge $A = \frac{1}{2}B(-y, x)$, the quantum mechanical Hamiltonian of this system can be written as [4]

$$H = \sum_{i=1}^{N} (-4\partial_i\bar{\partial}_i + z_i\partial_i - \bar{z}_i\bar{\partial}_i + z_i\bar{z}_i)$$
$$+ 4\eta \sum_{i\neq j}^{N} \left(\frac{1}{z_{ij}}\left(\bar{\partial}_i - \frac{z_i}{2}\right) - \frac{1}{\bar{z}_{ij}}\left(\partial_i + \frac{\bar{z}_i}{2}\right)\right) + 4\eta^2 \sum_{ij\neq i, i\neq k}^{N} \left(\frac{1}{z_{ij}\bar{z}_{ik}}\right) \quad (1)$$

[1] The Abdus Salam International Centre for Theoretical Physics, P.O. Box 586, Trieste, Italy.
[2] Permanent address: Laboratoire de Physique Théorique (LPT-ICAC), Faculté des Sciences, B.P 1014, Rabat, Morocco; e-mail: oudrhiri@fsr.ac.ma.

0020-7748/00/0200-0373$18.00/0

where z_i denotes the ith position of the electron, $z_{ij} = z_i - z_j$, $\partial_i = \partial/\partial_{z_i}$, and η is an odd integer value. In Eq. (1), the first term represents the quasicanonical momentum $\pi = P - A$, the second term describes the two-body interaction between the electrons, and the third term corresponds to the three-body interactions. Note that these interactions can be seen as a gauge potential.

It is convenient to define the creation and annihilation operators a_i^+ and a_i as follows:

$$
\begin{aligned}
a_i &= \frac{1}{2}\left(-2\partial_i + \bar{z}_i - 2\eta \sum_{j\neq i}^{N} \frac{1}{z_{ij}}\right) \\
a_i^+ &= \frac{1}{2}\left(2\bar{\partial}_i + z_i - 2\eta \sum_{j\neq i}^{N} \frac{1}{\bar{z}_{ij}}\right)
\end{aligned}
\tag{2}
$$

which satisfy the commutation relation

$$
[a_i, a_j^+] = \delta_{ij} \tag{3}
$$

We ignore the term $\Sigma_{j\neq i}^{N} \delta(z_i - z_j)$, which comes from the action of the operator ∂_i on $\Sigma_{j\neq i}^{N} (1/z_{ij})$, since it does not influence the derivation of the wave functions.

In terms of the creation and annihilation operators, the Hamiltonian (1) takes the form

$$
H = \sum_{i=1}^{N} (a_i^+ a_i + a_i a_i^+) \tag{4}
$$

Now let us investigate the ground state of this Hamiltonian. For this, we solve the equation of the ith electron,

$$
a_i \psi_i = 0 \tag{5}
$$

which leads to

$$
\left(-2\partial_i + \bar{z}_i - 2\eta \sum_{j\neq i}^{N} \frac{1}{z_{ij}}\right)\psi_i = 0 \tag{6}
$$

After some elementary manipulations, we find

$$
\psi_i = \prod_{j\neq i}^{N} (z_i - z_j)^{\eta} \exp\left(-\frac{1}{2} z_i \bar{z}_i\right) f(\bar{z}_i) \tag{7}
$$

where $f(\bar{z}_i)$ is an arbitrary holomorphic function on $\bar{z}$. Therefore, the system under consideration has many ground states which differ from one another by a given $f(\bar{z}_i)$ function. Now, for N electrons, we can define the total ground state by the product of N copies of the one-electron ground state, such that

$$\psi_\eta = \prod_{i,j\neq i}^{N} (z_i - z_j)^\eta \exp\left(-\frac{1}{2}\sum_i^N z_i\bar{z}_i\right) f(\bar{z}_i) \tag{8}$$

From the latter equation, it is easy to see that when the function $f(\bar{z}_i)$ is assumed to be constant, we recover the famous Laughlin wave function, namely

$$\psi_\eta^{\mathrm{L}} = \prod_{i,j\neq i}^{N} (z_i - z_j)^\eta \exp\left(-\frac{1}{2}\sum_i^N z_i\bar{z}_i\right) \tag{9}$$

which describes very accurately the ground state corresponding to the filling factor $\nu = 1/\eta$ (η odd integer value) characterizing the fractional quantum Hall effect.

Having obtained the ground-state wave functions, let us derive the corresponding excitation states. We recall that these states are built by the action of a_k^+ creation operators on the ground-state wave functions (9),

$$\phi_{k,\eta} = (a_k^+)\psi_\eta \tag{10}$$

where $k = 1, 2, \ldots, N$.

From Eq. (9) and with the help of the formula defining the a_k^+ operator, we obtain for the first excitation state the following expression:

$$\phi_{k,\eta} = \left(z_k - \eta\sum_{l\neq k}^{N}\frac{1}{\bar{z}_{kl}}\right)\psi_\eta \tag{11}$$

At this point we note that we can obtain the excitation state corresponding to N electrons. In this case, we define the total excitation state by the product of N copies of the one-electron excitation state

$$\phi_\eta = \prod_{k=1}^{N}\left(z_k - \eta\sum_{l\neq k}^{N}\frac{1}{\bar{z}_{kl}}\right)\psi_\eta \tag{12}$$

In this brief paper, we investigated the ground state of a system of electrons described by a Hamiltonian including electrons subject to a strong perpendicular magnetic field supplemented by two-body and three-body interactions. In a particular case, we showed that our ground states correspond to the Laughlin state, which is nothing but a ground-state wave function of the fractional quantum Hall effect at the filling factor $\nu = 1/\eta$. We also derived the excitation wave functions corresponding to the model under consideration.

ACKNOWLEDGMENTS

This work was done in part at the Abdus Salam International Centre for Theoretical Physics, Trieste, Italy. I thank Profs. A. El Hassouni, G.

Baskaran, and L. Dingping for valuable discussions and comments. It is a pleasure to thank Prof. Yu Lu for his help. I also thank Dr. M. Daoud for his reading of the manuscript.

REFERENCES

1. D. C. Tsui, H. L. Stormer, and A. C. Gossard, *Phys. Rev. Lett.* **48** (1982) 1559.
2. P. E. Prange and S. M. Girvin, *The Quantum Hall Effect*, Springer, New York (1990).
3. R. B. Laughlin, *Phys. Rev. Lett.* **50** (1983) 1395.
4. R. K. Ghosh and S. Rao, *Int. J. Mod. Phys. B* **12** (1998) 1125.